

Thermal Transport Properties of Graphene-Based F|S|F Junctions

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(Dated: 1 November 2011)

We present an investigation of heat transport in gapless graphene-based Ferromagnetic /singlet Superconductor/Ferromagnetic (FG|SG|FG) junctions. We find that unlike uniform increase of thermal conductance vs temperature, the thermal conductance exhibits intensive oscillatory behavior vs width of the sandwiched s -wave superconducting region between the two ferromagnetic layers. This oscillatory form is occurred by interference of the massless Dirac fermions in graphene. Also we find that the thermal conductance vs exchange field h displays a minimal value at $h/E_F \simeq 1$ within the low temperature regime where this finding demonstrates that propagating modes of the Dirac fermions in this value reach at their minimum numbers and verifies the previous results for electronic conductance. We find that for thin widths of superconducting region, the thermal conductance vs temperature shows linear increment *i.e.* $\Gamma \propto T$. At last we propose an experimental set-up to detect our predicted effects.

I. INTRODUCTION

In the recent years monoatomic graphite layers and structures containing this layers has attracted so much attentions theoretically and experimentally to itself¹⁻¹². The monoatomic graphite layer is called graphene and naturally treated as a two dimensional system. This two dimensional artificial system made by Novoselov *et al*^{13,14}. In graphene, low-excitation electrons follow the Dirac equation for their behaviors in various conditions and consequently Dirac equation can predict properties of the system¹⁵. Graphene exhibits very interesting properties that from one side confirm some of the predicted phenomena in relativistic quantum mechanics *e.g.* Klein's paradox and from other side its high mobility and controllable Fermi energy in experiment make it very interesting and suitable in laboratory and industry^{1,18,19}. From application point of view, it is very important to know the transport properties (charge, spin and thermal transport properties) of the devices including graphene junctions^{17,20-24}.

Thermal and charge transport properties are so much related to each other. Charge conductivity of the normal-superconductor (N/S) junctions at first was discussed by Blonder, Tinkham, and Klapwijk (BTK)²⁵ theoretically and their results could show good consistence with experiments. B-T-K take into account the contribution of the Andreev reflection³⁴ in the electronic transport of N/S junctions and use Bogoliubov-de Gennes (BdG) formalism to obtain the charge conductance at low temperatures. The BTK theory is limited to the clean regime of the heterojunctions, while cases with high impurities are

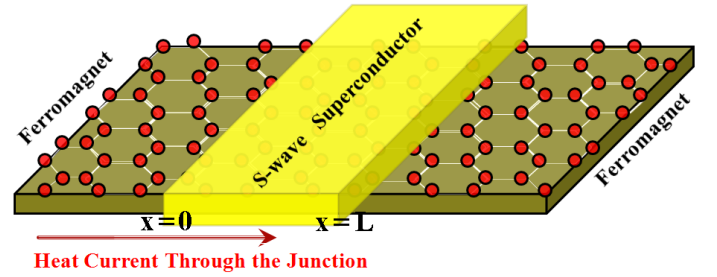


FIG. 1. (Color online) Schematic model for a system of the sandwiched graphene-based s -wave superconducting layer with width L , between two uniform graphene-based ferromagnetic substrates (FG/SG/FG). The ferromagnetic layers have similar directions in both sides.

not within the regime of validity. Bardas *et al.* and Devyatov *et al.*^{26,27} generalized the BTK model for the charge transport and calculated the thermal current through the N/S junctions. For metals at low temperatures the thermal conductivity Γ has a linear behavior with respect to temperature *i.e.* $\Gamma \propto T$, and their electronic conductivity reaches to constant value. So the Wiedemann-franz law is satisfied for them²⁸. By depositing a superconducting electrode on a graphene substrate, the superconducting correlations can leak into the graphene, due to the proximity effect³. Also ferromagnetism can be induced into the graphene by doping or using an external field^{29,30,32}.

In this paper we utilize the Dirac Bogolubov de-Genne equation and solve it for two dimensional systems including FG/SG and FG/SG/FG graphene junctions in which SG stands for the graphene-based s -wave superconductor and FG stands for graphene-based ferromagnetic substrate. We use the generalized BTK formula for heat transport through the junctions and investigate the thermal transport properties of the FG/SG and FG/SG/FG junctions and as a especial case, we assume the exchange field h , equal to zero for approaching to normal case *i.e.*

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NG/SG and NG/SG/NG. We find that the thermal conductance of the junction exhibits an *intensive oscillatory shape*, damping simultaneously by increasing the width of the superconducting layer. We find that the thermal conductance shows an exponential increase vs. temperature for large widths of superconducting layer that reflects the s-wave symmetry of the Dirac fermions inside the graphene as was mentioned by BTK but *we find that for thin widths of superconducting layer within the tunneling regime, thermal conductance Γ is linearly proportional to temperature T i.e. $\Gamma \propto T$* . Also we find that the thermal conductance vs. strength of the exchange field of the ferromagnetic substrate has a minimum near $h \simeq E_F$ which this value moves toward smaller values $h < E_F$ by increasing the temperature.

II. THEORY

We consider a graphene-based ferromagnetic/superconductor junction FG/SG which is placed in the xy -plane, and ideal interfaces between ferromagnet and superconductor located at $x = 0$, are perpendicular to the x -axis. For investigating properties of the mentioned system, one should solve the Dirac-Bogoliubov-de Gennes equation by following the Refs.^{3,33}

$$\begin{pmatrix} H_0 - \sigma h & \Delta \\ \Delta^* & -(H_0 - \bar{\sigma} h) \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix} = \epsilon_\sigma \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix}, \quad (1)$$

in which $H_0(\mathbf{r}) = -i\hbar v_F(\sigma_x \partial_x + \sigma_y \partial_y) + U(\mathbf{r}) - E_F$, where σ_x and σ_y are 2×2 Pauli matrices. Also, Δ stands for order parameter of the superconducting layer, and one can mention the order parameter of the system under consideration as $\Delta(\mathbf{r}, T) = \Delta(T)\Theta(x)$, in which $\Theta(x)$ is the well known step function, ϵ_σ is the excitation energy of the Dirac fermions with respect to the Fermi level, $h(\mathbf{r}) = h_0\Theta(-x)$ is the exchange field energy of the ferromagnetic layer. Here $\sigma = \pm 1$ stands for spin-up and -down quasiparticles, and $\bar{\sigma} = -\sigma$. $U(\mathbf{r})$ shows the Fermi mismatch vector(FMV), and since through out the paper we investigate heavily doping cases, we assume a large value for mismatch potential in comparison with the Fermi energy E_F within the ferromagnetic region i.e. $U(\mathbf{r}) = -\mathbf{U}_0\theta(\mathbf{x})$ and $U_0 \gg E_F$. Both of the u_σ and $v_{\bar{\sigma}}$ include two components of sublattices in hexagonal lattice of graphene. Thus each spinor in Eq.(1) involves four components, and for the electron-like excitations in the ferromagnetic region ($x < 0$) we obtain:

$$\psi_{e,\sigma}^\pm(x, y) = \frac{1}{\sqrt{\cos \alpha_\sigma}} e^{(\pm i k_{e,\sigma} x + i q y)} \begin{pmatrix} 1 \\ \pm e^{\pm i \alpha_\sigma} \\ 0 \\ 0 \end{pmatrix}, \quad (2)$$

and for hole-like quasiparticles:

$$\psi_{h,\bar{\sigma}}^\pm(x, y) = \frac{1}{\sqrt{\cos \alpha_{\bar{\sigma}}}} e^{(\pm i k_{h,\bar{\sigma}} x + i q y)} \begin{pmatrix} 0 \\ 0 \\ 1 \\ \mp e^{(\pm i \alpha'_{\bar{\sigma}})} \end{pmatrix}, \quad (3)$$

where $k_{e(h),\sigma}$ is component of the electron (hole)-like wave-vector, perpendicular to the interface and q is parallel component of the wave vector which remains conservative during the scattering process. The above appeared factors $1/\sqrt{\cos(\alpha)}$ and $1/\sqrt{\cos(\alpha')}$ guarantee same particle current transport by the four wave-functions³. Also $\alpha_\sigma(\alpha'_{\bar{\sigma}})$ are injection angles of electron (hole)-like quasiparticles with respect to the axis normal to the interface (x -axis). They are defined as:

$$\alpha_\sigma = \arcsin \left(\frac{\hbar v_F q}{\epsilon + E_F + \sigma h} \right), \quad (4)$$

$$\alpha'_{\bar{\sigma}} = \arcsin \left(\frac{\hbar v_F q}{\epsilon - E_F + \sigma h} \right), \quad (5)$$

$$k_{e,\sigma} = \frac{\epsilon + E_F + \sigma h}{\hbar v_F} \cos \alpha_\sigma, \quad (6)$$

$$k_{h,\bar{\sigma}} = \frac{\epsilon - E_F - \sigma h}{\hbar v_F} \cos \alpha'_{\bar{\sigma}}. \quad (7)$$

In the superconductor region ($x > 0$), wave function for the hole-like quasiparticles read as:

$$\psi_{S,h}^\pm = e^{i(\mp(k_0 - i\chi)x + qy)} \begin{pmatrix} e^{-i\beta} \\ \mp e^{-i(\beta \mp \gamma)} \\ 1 \\ \mp e^{(-i\gamma)} \end{pmatrix}, \quad (8)$$

and for the electron-like quasiparticles:

$$\psi_{S,e}^\pm = e^{i(\pm(k_0 + i\chi)x + qy)} \begin{pmatrix} e^{i\beta} \\ \pm e^{i(\beta \pm \gamma)} \\ 1 \\ \pm e^{i\gamma} \end{pmatrix}, \quad (9)$$

where

$$\beta = \begin{pmatrix} \cos^{-1}(\frac{\epsilon}{\Delta_0}), \epsilon < \Delta_0 \\ -i \cosh^{-1}(\frac{\epsilon}{\Delta_0}), \epsilon > \Delta_0 \end{pmatrix}, \quad (10)$$

$$k_0 = \sqrt{(\frac{U_0 + E_F}{\hbar v_F})^2 - q^2}, \quad (11)$$

$$\chi = \frac{U_0 + E_F}{k_0(\hbar v_F)^2} \sin \beta, \quad (12)$$

$$\gamma = \arcsin \frac{\hbar q v_F}{U_0 + E_F}, \quad (13)$$

here v_F is energy-independent Fermi velocity in graphene. We define right going ($+x$ -direction) and left going ($-x$ -direction) quasi-particle wave-functions with plus and minus signs e.g. ψ^+ , ψ^- respectively. In the mean field approximation, we assume high doping regime $U_0 + E_F \gg \Delta_0$ ³. It is clear that during the scattering process from the interface, q component of parallel wave vector and the energy of quasiparticles are constant (elastic scattering). The wave functions of the moving quasiparticles must satisfy the boundary conditions at the interface between ferromagnet and superconductor as in Ref.³. For FG/SG junction the boundary condition reads as:

$$\psi_{e,\sigma}^+ + r_{N,\sigma} \psi_{e,\sigma}^- + r_{A,\sigma} \psi_{h,\bar{\sigma}}^- = t_{e,\sigma} \psi_{S,e}^+ + t_{h,\sigma} \psi_{S,h}^-. \quad (14)$$

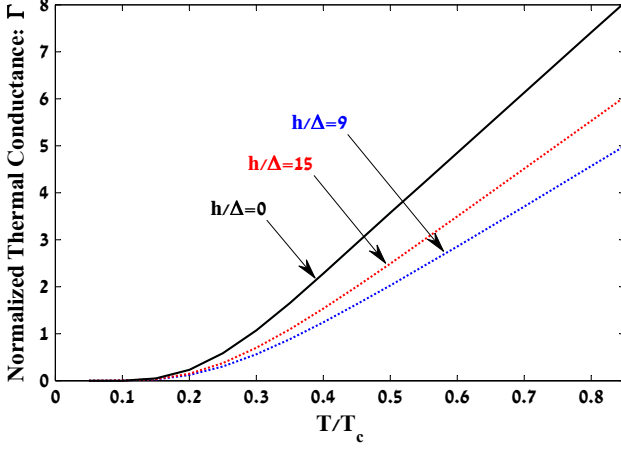


FIG. 2. (Color online) The normalized thermal conductance of single FG/SG junction vs temperature for three values of the ferromagnetic sheet's exchange field.

For the FG/SG/FG structure that schematically is shown in Fig. 1, however the boundary conditions at $x = 0$ and $x = L$ respectively are:

$$\psi_{e,\sigma}^+ + r_{N,\sigma} \psi_{e,\sigma}^- + r_{A,\sigma} \psi_{h,\sigma}^- = a_1 \psi_{S,e}^+ + a_2 \psi_{S,h}^- + a_3 \psi_{S,e}^- + a_4 \psi_{S,h}^+, \quad (15)$$

and

$$t_{e,\sigma} \psi_{e,\sigma}^+ + t_{h,\sigma} \psi_{h,\sigma}^- = a_1 \psi_{S,e}^+ + a_2 \psi_{S,h}^- + a_3 \psi_{S,e}^- + a_4 \psi_{S,h}^+. \quad (16)$$

Here, $r_{A,\sigma}$ is amplitude of the Andreev reflection, $r_{N,\sigma}$ is amplitude of the normal reflection, $t_{e,\sigma}$ and $t_{h,\sigma}$ are amplitudes of the electron-like and hole-like quasiparticle's transmission, respectively. Substituting the wave functions into the above boundary conditions, we calculate the coefficients in Eq. (14), for FG/SG and FG/SG/FG junctions. Then we now are able to obtain the probability of the Andreev reflection ($R_{A,\sigma} = |r_{A,\sigma}|^2$) and normal reflection ($R_{N,\sigma} = |r_{N,\sigma}|^2$). As seen in Fig. 1 interfaces are normal to the x -axis and $x = L$, so the x -dependent order parameter can be written as $\Delta(x) = \Delta_0 \theta(x) \theta(L - x)$.

The normalized thermal conductance $\Gamma = \Gamma' / \Gamma_0$ is given as follow^{26,31}:

$$\Gamma' = \Gamma_0 \sum_{\sigma=\uparrow\downarrow} \int_0^\infty \int_{-\pi/2}^{\pi/2} dE d\alpha_\sigma \cos(\alpha_\sigma) \{1 - |r_{N,\sigma}(E, \alpha_\sigma)|^2 - |r_{A,\sigma}(E, \alpha_\sigma)|^2\} \frac{E^2}{T^2 \cosh^2(\frac{E}{2T})}, \quad (17)$$

where $\Gamma_0 = E_F / 2\pi^2 \hbar^2 v_F k_B \Delta_0$ is a constant. Here all parameters are normalized, i.e. energies, with respect to pair potential at zero temperature ($\Delta_0 \equiv \Delta(T = 0)$), and temperatures, with respect to the critical temperature of superconducting order parameter (T_c). Throughout the paper we set $\Delta_0 = \hbar = k_B = 1$ in our computations.

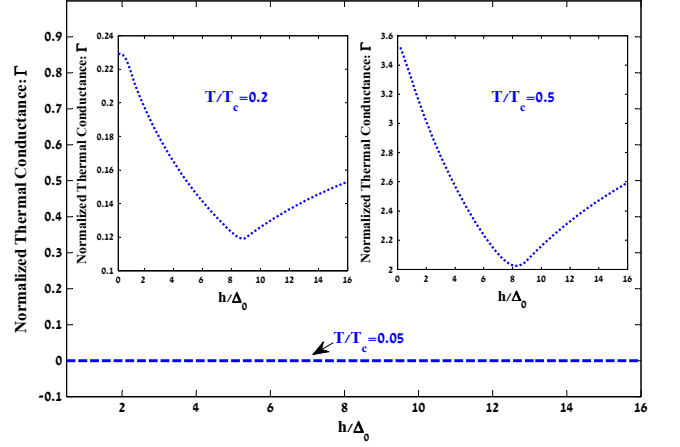


FIG. 3. (Color online) The normalized thermal conductance of the single FG/SG junction vs exchange field of the ferromagnetic layer for three values of temperatures.

III. RESULTS AND DISCUSSION

Now we proceed to present the main results of the paper. In order to obtain more realistic experimental results, we consider highly doping junctions and set a large mismatch potential U_0 . The Fermi energy of the graphene is externally controllable and can be tuned^{1,3}. Energy of quasiparticles remains within our regime of validity, for values near 0 to 1eV i.e. the quasiparticles follow the Dirac equation. Throughout the paper we fix the Fermi energy $E_F = 10\Delta_0$ which typically places Fermi energy within 10-15 meV. We change the width of the junction from $L \simeq \xi_S$ up to $L \simeq 9\xi_S$ and investigate how the thermal conductance is varied by the variation in L/ξ_S . Also we investigate other possibilities of variations in every parameter involved the problem and how they influence the thermal conductance. When we need fixed temperature, use $T/T_c = 0.2$, when we need fixed exchange field use $h/\Delta_0 = 8$ and when we need a fixed width, typically use $L/\xi_S = 4$. As will be discussed in the following sections, we find that unlike the exponential increase of the normalized thermal conductance with respect to the temperature in the FG/SG junctions reflecting the s -wave superconducting correlation, the normalized thermal conductance for FG/SG/FG junctions shows a linear increasing with respect to the temperature for small widths ($L \simeq \xi_S$) of the superconducting region i.e. $\Gamma \propto T$. For FG/SG/FG junctions, the thermal conductance vs. width of the superconducting region L/ξ_S shows an intensive oscillatory behavior, also it has a minimum vs. strength of the exchange field in both semi-infinite ferromagnetic graphene sheets.

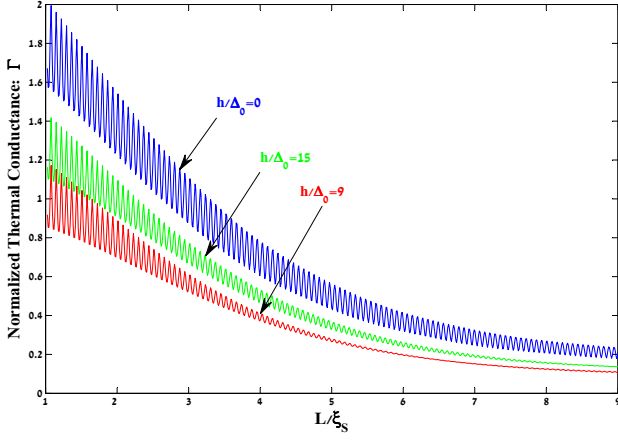


FIG. 4. (Color online) The normalized thermal conductance Γ vs the normalized width L/ξ_S of graphene-based superconducting layer for different values of h in FG/SG/FG junction. The temperature is fixed at $T/T_c = 0.2$.

A. Heat transport through the single Ferromagnetic/Superconductor junction

At first, we investigate how the normalized thermal conductance of the single junctions (FG/SG) behavior by changing both the temperature and exchange field h of the semi-infinite ferromagnetic sheet. We fix the Fermi levels of superconductor and ferromagnetic sheet at $E_F = 10\Delta_0$ and use a large mismatch potential U_0 , thus with such parameters we remain in the heavily doped regime. As seen in Fig. 2, the thermal conductance increases exponentially by increasing the temperature T , that verify the presence of the induced s-wave correlation in the graphene sheet. As seen in Fig. 2, the curve of the thermal conductance for $h/\Delta_0 = 15$ is placed between that of the thermal conductance for $h/\Delta_0 = 0$ and 9. This findings show that the thermal conductance has a minimum vs. h/Δ_0 as seen in Fig. 3. In Fig. 3 the normalized thermal conductance reach to a minimum near $h \simeq E_F$. Therefore by increasing the exchange field of the ferromagnetic layer, propagating modes of the Dirac Fermions decay, and reach to a minimum number near Fermi level. By increasing the temperature, the order parameter of the superconducting correlations decays, thus increasing of the temperature helps the exchange field to make propagating modes reach to their minimum value at smaller values of exchange field h as seen in Fig. 3. Now we proceed to present our findings for double FG/SG/FG junctions.

B. Heat transport through the double Ferromagnetic/Superconductor/Ferromagnetic junction

Now we turn our attention to the second graphene-based structure, namely double FG/SG/FG junction.

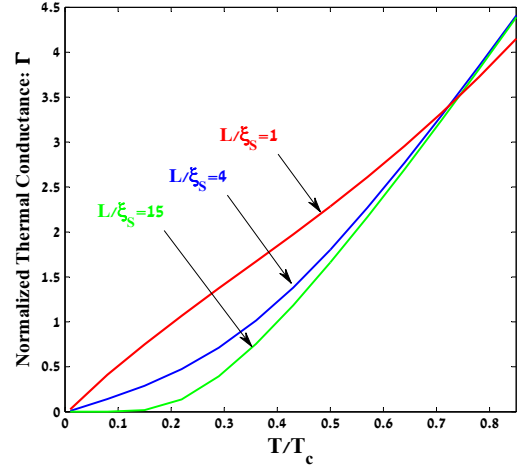


FIG. 5. (Color online) The normalized thermal conductance Γ vs temperature for three values of normalized widths of the superconducting layer $L/\xi = 1, 4, 15$ in FG/SG/FG junction. The exchange field is fixed at $h/\Delta_0 = 8$.

The suggested set-up of the junction schematically is shown in Fig. 1. Such junctions can be made by depositing a superconducting electrode on top of the graphene sheet. As before we fix the Fermi energy of the superconductor and ferromagnetic at $E_F = 10\Delta_0$. As shown in Fig. 4, the normalized thermal conductance shows intensive oscillatory behavior versus width of superconductor region. This manner is a consequence of the coherent interference of the Dirac fermions in facing with two interfaces. Also, by increasing the width of superconducting region, the normalized thermal conductance loses the amplitude of its oscillation and shows exponentially decrement towards single junction. When the Dirac fermions lose their coherency, they show the mentioned manner and this illustrates the fact that when the width of the superconducting region becomes larger, the probability of tunneling of the Dirac fermions across the superconducting region is reduced. The thermal conductance versus temperature is shown in 5. The thermal conductance for small widths of the superconducting region shows a linear behavior similar to the behavior of the thermal conductance of metals, discussed in the introduction. This manner reflects the tunneling process of the Dirac fermions through the superconducting region. Thus one can expect that when the width of this region goes to larger values, this phenomena (tunneling) is reduced and the thermal conductance behaves like the single junction which is clearly shown in Fig. 5 for $L/\xi_S = 15$. The thermal conductance as a function of the exchange field of the ferromagnetic region is plotted in Fig. 6. The behaviors of the thermal conductance vs. h for double junction (FG/SG/FG) and the single junction (FG/SG) are the same. The thermal conductance reach to a minimum near $h \simeq E_F$, and the minimum value is moved to smaller ones by the temperature increment.

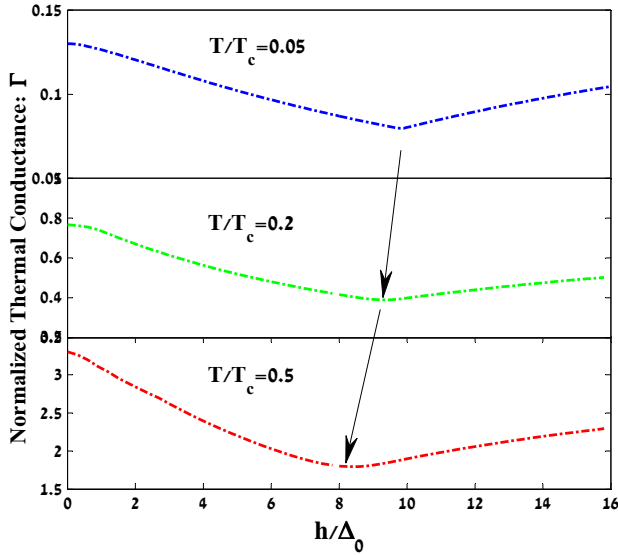


FIG. 6. (Color online) The normalized thermal conductance Γ vs strength of normalized exchange field h/Δ_0 of the ferromagnetic sides for three values of temperatures in FG/SG/FG junction. The width of the superconducting layer is fixed at $L/\xi_S=4$.

IV. SUMMARY

In the present paper we have considered two types of gapless graphene-based junctions, FG/SG and FG/SG/FG with s -wave superconducting electrodes, and we have investigated the heat transport properties of the systems. In particular, we have investigated how the variations of the variable quantities in the system can influence the thermal conductance of the junctions. Utilizing the Dirac-Bogoliubov de Gennes equation for quasiparticles inside the graphene and appropriate boundary conditions for the obtained wave functions, we have derived the Andreev and normal reflection coefficients, and used them for calculating the normalized thermal conductance numerically. We found that for single junction (F/S) the normalized thermal conductance indicates the previous exponential form vs. temperature. Increasing exchange field h lowers the propagating modes of the Dirac Fermions down to a minimum value. Due to the tunneling phenomena of the Dirac fermions through the superconducting layer, the heat transport properties of the double F/S/F junctions are different from single F/S junction. The thermal conductance vs temperature exhibits linear behavior in the tunneling limit (small widths of the superconducting region). The thermal conductance vs. width of the superconducting region exhibits very intensive oscillatory behavior and also the quantity indicates previous behavior of the single junction (F/S) vs. h *i.e.* the enhancement of the exchange field from 0 up to a value near E_F , lowers the propagating modes of the Dirac fermions.

ACKNOWLEDGMENTS

The authors appreciate very useful and fruitful discussions with Jacob Linder. The authors would like to thank the Office of Graduate Studies of Isfahan University.

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